

though there is no fundamental reason why this should not be improved.

#### Reference

- <sup>1</sup> Persson, S. L., "Method for determination of velocity distribution in a thin film," AIAA J. 2, 372-373 (1964).
- <sup>2</sup> Jeffrey, R. C., "Particle motion in Poiseuille flow," Ph.D. Thesis, Univ. of Cambridge, (1964).
- <sup>3</sup> Nedderman, R. M., "Velocity profiles in thin liquid layers," Ph.D. Thesis, Univ. of Cambridge (1960); available in microfilm from Micro Methods Ltd., East Ardsley, Wakefield, Yorks, England.
- <sup>4</sup> Nedderman, R. M., "The use of stereoscopic photography for the measurement of velocities in liquids," Chem. Eng. Sci. 16, 113 (1961).
- <sup>5</sup> Wilkes, J. O. and Nedderman, R. M., "The measurement of velocities in thin films of liquid," Chem. Eng. Sci. 17, 177 (1962).

### Reply by Author to R. M. Nedderman

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NEDDERMAN observes that the main assumption in my method concerning the uniform distribution may be doubtful. This argument was based on experiments done by Jeffrey in full tube Poiseuille flow. According to the comment, Jeffrey's experiments have shown that migration may occur. The physical reasons for this migration are, however, not clear to the author from the comment, and it is thus difficult to imagine what kind of effect one would expect in thin water films.

From the velocity profiles given in Ref. 1 it may be noticed that  $dv/d\delta$  approaches zero at  $\delta = 0$ . This may be regarded as supporting Nedderman's comment. In profiles measured at lower mean film velocities (and thus thicker films), this effect does not occur, however, as can be observed from Fig. 1.

Another possible explanation for the slope of the profiles given in Ref. 1 might be slip flow due to the use of particles that are too big compared to the film thickness. The results given in Fig. 1 were obtained with particles of the same size but with films of the order of five times the thickness of the

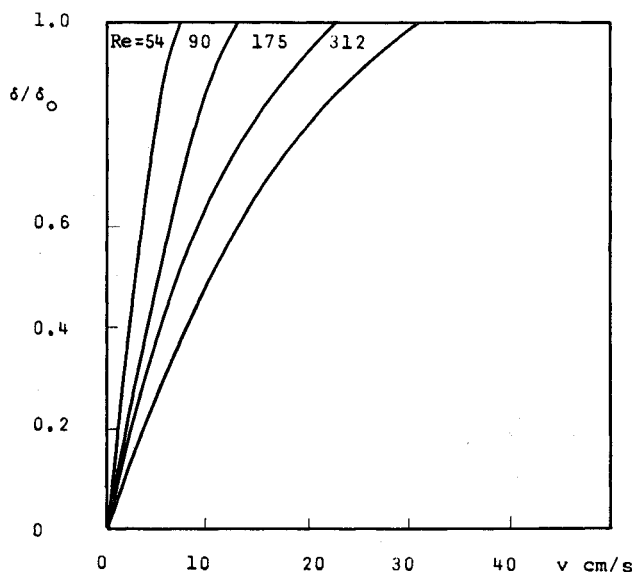


Fig. 1 Influence of Reynolds number on velocity profiles without concurrent air flow.

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previously studied rapid films. However, this eventual slip flow effect needs further investigation.

#### Reference

- <sup>1</sup> Persson, S. L., "Method for determination of velocity distribution in a thin liquid film," AIAA J. 2, 372-373 (1964).

### Erratum: "Comment on 'Equilibrium Orientations of Gravity-Gradient Satellites'"

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[AIAA J. 2, 1357 (1964)]

IN the above Technical Comment, Eq. (3) should read

$$C\ddot{\gamma} + \Omega^2(A - B)\gamma - \Omega(A - B - C)\dot{\beta} = 0$$

The symbol  $\ddot{\gamma}$  dropped out after final page proofs had been released.

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### One-Dimensional Rayleigh Flow of a Partially Ionized Gas

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#### Nomenclature

$\rho$	= gas density $nm$ , where $n$ is the total number of heavy particles per cubic centimeter, i.e., the number density of ions and neutral atoms (not the number density of neutral atoms alone as suggested by Yen)
$A$	= area of the channel
$u$	= gas velocity in the channel
$q$	= energy supplied per cubic centimeter per second to the flowing gas
$m$	= mass of a neutral atom
$m_e$	= mass of an electron
$\alpha$	= degree of ionization
$T$	= absolute static temperature of the flowing gas
$X$	= ionization potential of the neutral atom
$p$	= static pressure of the flowing gas
$2\pi\hbar$	= Planck's constant
$h$	= specific enthalpy of the flowing gas
$(g_0)_a, (g_0)_i$	= ground state statistical weights of the neutral atom and first ion, respectively

IN a recent note Yen<sup>1</sup> has studied the Rayleigh flow of a partially ionized gas and concluded that "choking at Mach number equal to 1" cannot be obtained. The purpose of the present note is to demonstrate the incorrectness of the basic equations, and the conclusion, of Yen.

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The correct basic equations for the one-dimensional channel flow of a partially ionized gas with heat addition are

$$(d/dx)(\rho Au) = 0 \quad (1)$$

$$\rho u(du/dx) + (dp/dx) = 0 \quad (2)$$

$$\rho u(d/dx)[h + (u^2/2)] = q \quad (3)$$

$$\frac{\alpha^2 p}{1 - \alpha^2} = \frac{(2\pi m_e)^{3/2} k^{5/2}}{(2\pi \hbar)^3} \frac{2(g_0)_i}{(g_0)_a} T^{5/2} \exp - \left( \frac{X}{kT} \right) \quad (4)$$

$$p = \rho(1 + \alpha)(k/m)T \quad (5)$$

The specific enthalpy of a partially ionized monatomic gas is derived by Jones and McChesney<sup>2</sup> as

$$h = \frac{5}{2}(kT/m)(1 + \alpha) + (\alpha X/m) \quad (6)$$

with a term similar to Eq. (8). Namely, we can write Eq. (8) as

$$\frac{\gamma'}{\gamma' - 1} \frac{k}{m} nu \frac{d}{dx} [(1 + \alpha)T] \quad (13)$$

Here,  $\gamma' = h/e$ , where  $e$  is the specific internal energy, i.e.,

$$h = [\gamma' / (\gamma' - 1)](p/\rho)$$

which is analogous to the perfect gas relation. With the approximation of Eqs. (4) and (6), our expression for  $\gamma'$  becomes

$$\gamma' = \frac{5(1 + \alpha) + (2\alpha X/kT)}{3(1 + \alpha) + (2\alpha X/kT)} \quad (14)$$

If we solve Eqs. (1-5), then after considerable algebraic reduction we obtain

$$\left[ \frac{nkT}{m} \frac{1 + \alpha}{2 - \alpha} \left\{ \alpha(1 - \alpha) \left( \frac{5}{2} + \frac{X}{kT} \right)^2 + 5 \right\} - nu^2 \left\{ \frac{3}{2} (1 + \alpha) + \frac{\alpha(1 - \alpha)}{(2 - \alpha)} \left( \frac{3}{2} + \frac{X}{kT} \right)^2 \right\} \right] \frac{du}{dx} + \left[ \frac{nu kT}{m} \frac{1 + \alpha}{2 - \alpha} \left\{ \alpha(1 - \alpha) \left( \frac{5}{2} + \frac{X}{kT} \right)^2 + 5 \right\} \right] \frac{1}{A} \frac{dA}{dx} - \frac{q}{m} = 0 \quad (15)$$

This equation is valid provided electronic excitation of the atomic and ionic species can be ignored and there is no multiple ionization. This approximation has also been made in the Saha equation, Eq. (4), where the species statistical weights replace the partition functions.

Combining Eqs. (3) and (6) gives the energy equation

$$nu^2 \frac{du}{dx} + \frac{5}{2} \frac{k}{m} nu \frac{d}{dx} [(1 + \alpha)T] + \frac{nu X}{m} \frac{d\alpha}{dx} = \frac{q}{m} \quad (7)$$

The energy equation used by Yen is similar in appearance to (7) except that he writes the second term on the left-hand side as

$$\frac{\gamma}{\gamma - 1} \frac{k}{m} nu \frac{d}{dx} [(1 + \alpha)T] \quad (8)$$

Here  $\gamma$  is the ratio of the specific heats which Yen takes as  $\frac{5}{3}$ . Furthermore, Yen introduces the sound speed  $a$  in the partially ionized gas as a whole as

$$a^2 = \frac{5}{3}(k/m)(1 + \alpha)T \quad (9)$$

Equations (8) and (9), which are Yen's, are incorrect for a partially ionized gas. The reasons are discussed in detail by Jones and McChesney where it is shown that, for a reacting gas mixture, the specific heat ratio  $\gamma$  does not characterize isentropic processes in the flow as it does for an ideal gas. In the paper just cited, it is shown that the correct expression for the equilibrium sound speed is

$$a^2 = \gamma^*(k/m)(1 + \alpha)T \quad (10)$$

where

$$\gamma^* = \frac{2\gamma}{(2 - \alpha)(1 + \alpha)} \quad (11)$$

$$\gamma = \frac{5(1 + \alpha) + \alpha(1 - \alpha^2)[\frac{5}{2} + (X/kT)]^2}{3(1 + \alpha) + [2\alpha(1 - \alpha)/(2 - \alpha)][\frac{5}{2} + (X/kT)]^2} \quad (12)$$

It is possible to characterize a real gas flow energy equation

Choking occurs when the numerator of the coefficient of  $du/dx$  vanishes; this gives  $u = a$ , where  $a$  is given by the foregoing Eqs. (9-11), i.e., we get choking at Mach 1.

This result would have been expected from the studies of Shercliff,<sup>3</sup> who has shown that the speed of sound is a critical velocity for any single phase fluid in equilibrium provided certain criteria are obeyed (which, for a partially ionized gas, are obeyed).

Equation (15) reduces to the usual equation for perfect gas flow when  $\alpha = 0$  and 1 where the usual relations between  $h$ ,  $\gamma$ ,  $p$ ,  $\rho$  and  $a$ ,  $\gamma$ ,  $p$ ,  $\rho$  hold. However, in the fully ionized case, the Saha equation loses its meaning.

## References

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## Reply by Author to M. McChesney

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IN my note,<sup>1</sup>  $n$  is meant to be the original number density of the neutral particles (i.e., when  $\alpha = 0$ ). Thus,  $n_n = (1 - \alpha)n$ ,  $n_e = n_p = \alpha n$ , and the total number of particles is  $n_n + n_e + n_p = (1 + \alpha)n$ . The total pressure is  $p = (1 + \alpha)nkT$ . Of course,  $n = n_n + n_p$ . The energy equation (7) given for monatomic gases in McChesney's comment is identically the same as the one given in Ref. 1, with  $\gamma$  taken to be  $\frac{5}{3}$ . Therefore, my basic equations are correct.

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